GUARDING OUR HIGH SCHOOL

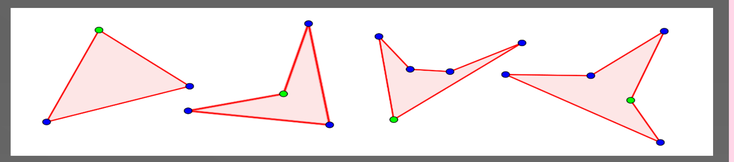
How can we guard our school grounds with the minimun possible number of cameras?

This problem arises from the need to monitor the valuable collections that had in art galleries with the least possible budget. It is monitored with cameras that rotate rotate 360º.

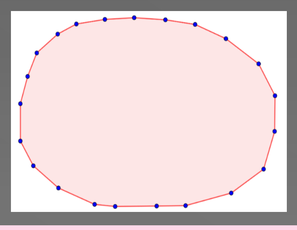
The art gallery problem was raised by Victor Klee in 1973.

The first solution was given by Chvátal, but, usually, it is used the solution given by Steve Fisk, in 1978:

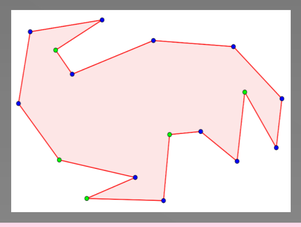
Theorem (Chvátal-Fisk, 1978): Given a polygon with n vertices and without holes, $\left\lfloor \frac{n}{3} \right\rfloor$ cameras are enough and sometimes necessary, to guard it.



The solution in polygons with 3, 4 and 5 vertices is very simple, since with a camera we can guard everything.



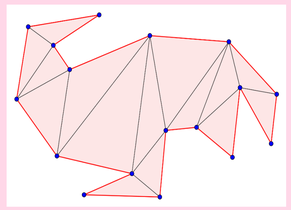
The solution in convex polygons is very easy too. We need just one camera.



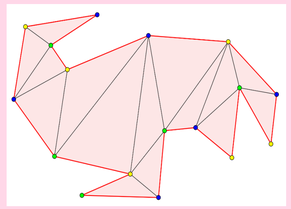
What happens in polygons with more than 5 vertices?

To guard this kind of polygons and get easy solutions we use the following method (given by Fisk):

1. Triangulate the polygon.

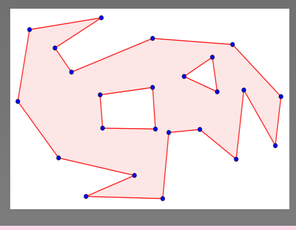


2. Colorate the vertices by choosing three colours so that in each triangle the three colours are.



3. The solution is to place the cameras in the verices of the colour that appear less.

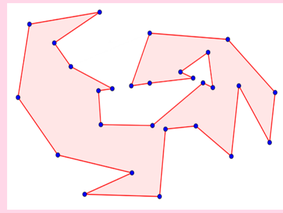
5 (= $\left\lfloor \frac{17}{3} \right\rfloor$) cameras must be place in green vertices.



What happens if we have a polygon with holes?

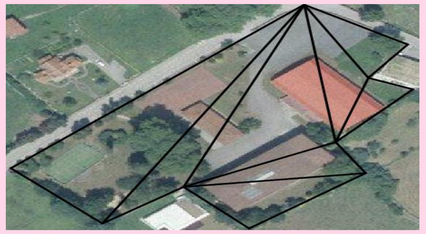
Theorem (Bjorling-Sachs, Souvaine, Hoffman, Kaufman, Kiegel, 1991): Given a polygon with n vertices and h holes, $\left\lfloor \frac{n+h}{3} \right\rfloor$ cameras are enough, and sometimes necessary, to guard it.

The idea to solve this new version is the same as before, but, firstly, we will have to do a kind of corridor to join the holes with the exterior part of the polygon.

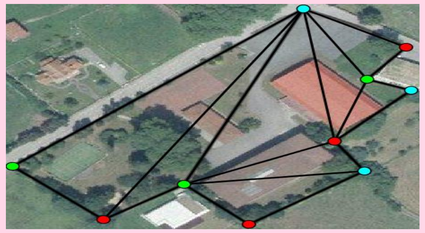


Our high school playground.

Solution without buildings (not real solution).
1-Triangulate.



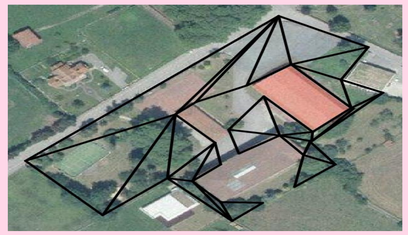
2-Colorate the vertices.



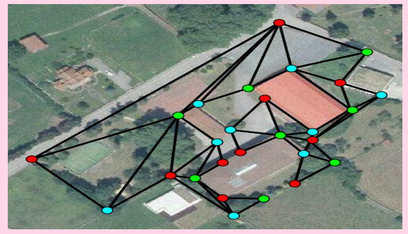
We must place 3 cameras in the blue or green vertices, but this solution is not real.

Solution with buildings (real solution).
1-Unite the buildings by create a hallway.

2-Triangulate.



3-Colorate the vertices.



We must place 8 cameras in the green vertices to guard our high school's playground.

Pablo Azpiazu González. Alba Escudero Revuelta. Cristina García Ríos. Miguel Gómez Rey. Alejandro López López.Sonia López Revuelta. Joana Oria Conde. Roberto Ortiz Martínez. Pablo Pardo García. Cristina Rivero Sainz. Sonia Ruíz Santos. Mark Schmuckle Rico. **Prof:** Mª Pilar Sabariego Arenas.